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Performance of expert abacus operators*

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Abstract

Ten expert abacus operators were given various restrictions and distractions during addition of ten numbers of 3–5 figures. All subjects except one could calculate very rapidly without an abacus, probably relying upon its mental representation. Some of those at an intermediate level of mastery moved their fingers as if they had been manipulating a real abacus, and prohibition of this movement or interfering finger-tapping reduced their performance. All the subjects could answer simple non-mathematical questions during abacus calculation without increasing time or errors, but answering extraneous mathematical questions was very hard.

We sought in this study to investigate, by distracting expert abacus operators in various ways, the cognitive and psycho-motor mechanisms involved in working with the modern Japanese version of this venerable instrument of calculation. Abacus operation is psychologically interesting as a typical over-learned cognitive skill or habit. Almost all Japanese children learn how to use the abacus in the third grade in school and have some practice until the 6th grade. In addition, there are many private training schools for children and adolescents. According to the Yomiuri Shimbun (May 13, 1971), one of the

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best-known newspapers in Japan, more than two million take some qualifying examination for one of the grades (10th class to 1st class; then level 1 master to level 10 master grade) every year. In some junior high schools and high schools, there is an abacus club, members of which compete in matches and tournaments. Usually these players practice at least a few hours every day for several years. This undoubtedly produces an overlearned skill. Since overlearning is characteristic of the routine use of an established skill, and since the nature of this final stage may be different from earlier stages of acquisition, the presence of overlearning increases the validity of generalizations from experimental studies of a skill to its daily use.

In daily observation, two things about abacus calculation as an overlearned skill are especially intriguing. First, abacus operation tends to be gradually interiorized to such a degree that many abacus masters can calculate even faster and more accurately without an abacus than with the instrument itself. They move their fingers during mental calculation as if they were actually fingering an abacus, though grand masters manage not to do so. Their finger movements are analogous to mouthing the words while reading. Secondly, while expert practitioners are using an abacus they can carry on a conversation. Like the subjects who did a pair of high ideomotor compatibility tasks in Greenwald's experiment (1972), they can do two things at one time – calculating and asking and/or answering questions. This suggests, hypothetically, that after their overlearning, abacus operators need only a limited amount of attention for calculation, and little linguistic processing is involved therein.

Since there have been almost no systematic, scientific studies on abacus operation, the experiment reported here was aimed at examining mental calculation and the effects of answering questions during operation under controlled experimental conditions relying mainly upon the dual task technique. Specifically, it concerned 1) whether or not most expert operators could calculate mentally, 2) how their performance would change if their finger movement was prohibited or interfered with, 3) whether or not they could answer a question asked during abacus operation and/or 4) during mental operation, and 5) how the answers to 1)–4) would depend upon their level of mastery.

Method

Procedure

The experiment was individually administered. Prior to the main part, the digit recitation sub-test from Wechsler Adult Intelligence Scale (WAIS) was given, first as prescribed in the manual (M1), and the second time by reading each series as if it had been a large number (e.g., 628 was read as six hundred twenty-eight, M2). Better performance was expected under the latter condition, because abacus operators surely have much experience of representing and processing orally given large numbers.

Seven series of 10 addition problems were given in turn. Each problem consisted of 10 three-to-five digit numbers, 40 digits in all. All digits were randomly chosen, with the restriction that the most significant digit should not be zero. The problems were printed out from the computer, ten numbers from top to bottom, with the corresponding figures in the same column and presented to the subjects visually. After two or three practice items of the same sort, a subject was required to give oral answers for each of the problems as quickly and as accurately as possible. Time was measured from the 'begin' signal to where she started her answer.

The following experimental conditions were set in order to examine effects of three independent variables on computation time and accuracy: availability of abacus (E1, E2; E3, E5); finger movement (E2, E6, E7) subsidiary questions (E1, E3, E4; E2, E5).

(E1) use of abacus as usual

No distractions nor restrictions were given.

(E2) mental calculation

No abacus but no distractions nor restrictions. A subject was permitted to write down partial answers during the calculation, but required to give the final answer aloud.

(E3) use of abacus interposed with a simple, non-mathematical question

A factual or preference question, such as "What is your name?", "What is the highest mountain in Japan?", "What is the capital of England?", "What is your favorite food?", was orally presented 5 seconds after the signal to begin calculation.

(E4) use of abacus interposed with a simple mathematical question

Five seconds after the 'begin' signal, a subject was given a simple mathematical question orally, e.g., "What is 3 plus 8?", "What is 16 minus 9?", "What is 72 divided by 6?"

(E5) mental calculation interposed with a non-mathematical question

A subject, denied the use of the abacus as in E2, was asked a non-mathematical question as in E3.

(E6) mental calculation without moving fingers

A subject was prohibited from moving her fingers to simulate abacus operation, but no distraction was presented.

(E7) mental calculation with compulsory tapping

A subject was requested to tap on the table with the left hand (problems 1–5), then the right hand (6–10) at her own preferred speed. She was permitted to move the fingers of the other hand, but no one was observed doing so.

Since those conditions under which subjects were distracted had been preceded by 'baseline' tasks (E1 and E2), there may have been contamination through warm-up effects, or, on the other hand, fatigue. However, daily observation suggests that performance of skilled operators is so stable over a period of a few hours, that in this test session of 30 to 35 minutes, we can ignore these factors with little risk.

Speed of calculation was compared between conditions and between subjects by 2 factorial ANOVA, treating each subject's solution times on 10 problems as 'independent' behavior samples from universe, because little inter-problem carry-over effect was expected to exist for speed. Accuracy was compared only between two conditions by the Wilcoxon matched pair signed rank test, using the number of correct responses of each subject. Thus, the number of pairs was reduced when some subjects made the same number of correct responses under the two conditions. Two-tailed test was used throughout for both speed and accuracy.

Subjects

Ten skilled female abacus users, 16–18 years of age, served as subjects. They were the ablest operators of the 'abacus club' of a senior high school and qualified as *ikkyu* (first class) or better. Six of them had acquired *shodan* (level 1 master) or an even higher grade. In order to be qualified as ikkyu, one has to be able to perform 2.4 single operations of addition or subtraction per second when a problem is presented visually, but is not required to calculate mentally. Qualifications above this, i.e., for master levels, require greater speed and some skills of mental operation.

Results

1. Most subjects said in a post-experiment informal interview that they imagined arranged beads of an abacus when they heard a number. However,

the digit memory span of abacus masters did not greatly exceed the national median. Range for M1 was 5 to 9 with a median of 6.5 and that for M2 5 to 8 with a median of 7. There was no marked correspondence between memory span and skill of abacus operation.

2. Means and standard deviations of response (calculation) time and the number of correct responses for each condition over individual subjects are shown in Table 1. Most subjects, working with the abacus at an average rate of two to four digits per second, made few, if any, errors, in E1. Stability within subjects was high, i.e., mean within-subject standard deviation over 10 problems was 1.68 sec.

Table 1. *Means and standard deviations (SD) of reaction time (RT) and number of correct responses (CR) over subjects*

	RT		CR	
	mean	SD	mean	range
E1, abacus	15.1	3.14	8.5	6–10
E2, mental	13.9	4.38	5.8	2–9
E3, abacus-nonmathematical	14.9	3.24	8.3	5–10
E4, abacus-mathematical	15.8	2.70	7.1	4–9
E5, mental-nonmathematical	14.2	3.75	3.0	1–5
E6, mental-fixing hand	13.3	4.15	3.5	0–9
E7, mental-tapping	14.3	5.10	3.8	0–7

Note: Data for one subject unable to calculate mentally without pencil and paper are omitted in E2, E5, E6 and E7.

3. There appeared very clear individual differences in mental calculation, E2, again with high stability within subjects (mean standard deviation was 1.38 sec). If we rank the subjects according to their speed on E2, half of them, all of intermediate capacity, moved their fingers as if to simulate abacus operation. S1, S2, S3 and S5 did not move their fingers. The weakest (S10) resorted to pencil and paper and her data for mental calculation are excluded. Mental calculation took significantly less time than actual operation for the best five. For the remaining four (excluding S10) the reverse was true. When 2 (E1, E2) × 9 (subjects 1–9) factorial ANOVA with 10 repetitions in each cell was conducted, the interaction effect was highly significant, $F(8, 262) = 12.8, p < 0.01$.

4. Though prohibition of hand movements did not produce any consistent change in the rate of calculation, performance deteriorated (between E2 and

E6, by the Wilcoxon matched pair signed rank test, $T = 4, n = 9, p < 0.025$). This means moving fingers helped at least some subjects. However, this, and even finger tapping, did not prevent the best two (S1 and S2) from scoring nearly 100% (90% each).

5. The negative effect of finger tapping was similar to that of mere immobilization of subjects' hands. It significantly reduced the number of correct responses (between E2 and E7, $T = 1, n = 9, p < 0.01$), but its overall effect on speed was not clear (F (1, 162) = 2.31, $p > 0.1$). Right hand tapping was not significantly more disturbing than left hand tapping (average calculation time among S1–S9 was 15.4 sec for right with 28 errors while 13.4 for left, with 29 errors).

6. With an abacus, being asked non-mathematical questions affected neither accuracy nor speed (between E1 and E3, for speed F (1, 180) = 0.457; for the number of correct responses, $T = 11, n = 7$). On the contrary, questions involving calculation had a clear inhibitory effect: they often completely blocked performance of the main task. This was the only condition under which our best two subjects needed significantly longer time. It seemed to be impossible for the subjects to calculate two sets of numbers at the same time. (Between E1 and E4, for speed F (1, 180) = 7.978, $p < 0.01$; for accuracy $T = 5, n = 10, p < 0.025$).

7. Without an abacus, asking non-mathematical questions had a negative effect on accuracy generally, and on speed only for the upper five subjects (between E2 and E5, for speed, F (1, 162) = 1.95, $p > 0.1$; for accuracy $T = 0, n = 9, p < 0.01$; for speed of S1–S5, F (1, 90) = 7.80, $p < 0.01$).

8. Though our subjects tried hard to answer all interfering questions, they answered several incorrectly and sometimes failed to respond at all. Excluding S10, who failed to reply on 18 items of the 30, their average number of no-response-plus-error was 4.9 (16.3%). Incidentally, they could not answer some questions even after the experiment was over.

Discussion

First, the problem of 'interiorization' will be discussed. All subjects but S10 showed extraordinary speed in mental calculation, and four of them (S1, S2, S4 and S5) performed with considerable accuracy (80% correct or better on E2). Although the fact that S10 relied solely upon the pencil-and-paper technique means that mastery of manual abacus operation does not necessarily guarantee drastic improvement in mental calculation, the other nine subjects seemed to have 'interiorized' their skill in abacus operation to a varied extent.

That their mental calculation derived from abacus operation was suggested by the imitative finger movements of the subjects of intermediate capacity (the very skilled subjects reported they had been moving fingers sometime before), and also by the detrimental effects of manual restriction and extraneous movement (E6 and E7). If their mental operations were purely cognitive, keeping their hands still would not have been inhibitory. Even finger tapping should produce no difficulty, since it is only an 'emission' task, according to Peterson's classification (1969), requiring neither discrimination nor transformation. In fact, Jastrow (1891–2) found no mutual interference between a cognitive task and rhythmic finger tapping when the latter was self-paced. However, E6 and E7 significantly reduced the accuracy of mental calculation, except for the best two subjects. In other words, abacus operation tends to interiorize into mental operation through a transition stage wherein the mental operation is not completely independent from the motor system and abacus-simulating finger movement gives important support. It is not clear why finger movements are essential for the transition to mental abacus operation, but they might provide feedback about the progress of the calculation, or serve as a representation of the digits and the calculation. This kind of dactylic coding has been observed in memory experiments with deaf children (Locke and Locke, 1971).

As they progress, the mental operation comes to involve only visual representation without relying upon the motor elements. In this 'mature' stage, neither the prohibition of finger movement nor tapping causes any disturbance. In addition, their mental calculation is much faster than their actual abacus operation, because it is no longer limited by their fingering speed. This exclusion of peripheral activity has also been reported in a study of exceptional skill in solving arithmetic problems by mental calculation using Western methods (Hunter, 1966). Longitudinal studies are necessary to establish that individuals must make these qualitative changes in the underlying processes if their performance is to improve and extend to mental abacus calculation. This would exclude the interpretation that each of these different processes is a stable characteristic of a subgroup of subjects and does not change as a necessary condition of improved performance. Factors inhibiting the interiorization for S10 were not analysed here, but certainly deserve close examination.

Secondly, from E3 and E4, the compatibility of additional conversational tasks with abacus operation and its limitation will be discussed. Answering a verbal query during calculations did not disturb manual operation (E3). This confirmed daily observation and strongly suggests that 1) their operations are now quasi-automatic and need only a limited amount of attention, and that 2) while conversation is carried out in the auditory-linguistic system, abacus operation is conducted mainly in the visual-motor system. These suggestions were also supported by an additional experiment showing that immediate repetition of a word or a series of 3 digits did not have any detrimental effect on computation time or accuracy. Moreover, magnitude of this compatibility is rather constant among the skilled practitioners, irrespective of their relative level of mastery.

Unlike abacus masters, people who have little or no experience in abacus operation seem to use the auditory-linguistic system in calculation (paper-and-pencil or mental) to a considerable extent. Therefore, even simple repetition of an extraneous word is expected to be incompatible with calculation. This was confirmed in another auxiliary experiment with three university students with minimal experience of abacus operation. Their calculation was significantly slowed down even by immediate repetition, which proved to have no effect on the expert abacus practitioners' calculation. Matching the subjects and experts for arithmetic ability, or the use of the same experts for both methods of calculation would strengthen these results.

However, the same combination as E3 was extremely difficult in E4. Even the expert abacus users, for whom abacus operation seems nearly automatic, could not calculate another set of numbers promptly while working on the abacus. This clearly established a limit to the generality of the daily observation. Two interpretations can be raised: first, calculation requires a considerable amount of attention or information-processing while answering factual or preference questions involves only little. A second possible interpretation takes the similarity or overlap of the two operations as critical. The expert abacus users may have relied upon a mental abacus even for the simple mathematical questions asked in E4. This point will be examined more systematically in varieties of dual-task experiments including those with more demanding non-mathematical questions.

No marked differential effects were observed in either condition between the best and poorest 5 subjects. This may be due to the fact that less advanced practitioners tend to waste a certain amount of motion or attention during calculation, while those who are more advanced work more efficiently so that little or no such 'empty time' is expended.

E5, concerned with the compatibility of conversation with mental operation, cast some light on the differences between abacus and mental operation. Considering that performance was disturbed in E5 while non-mathematical questions did not deteriorate abacus operation (E3), mental calculation seems less stable than abacus operation. Mental calculation may be so demanding that it is disrupted by the modest additional processing posed by non-mathematical questions.

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Résumé

A dix opérateurs experts d'abaque sont donnés de diverses restrictions et des distractions pendant l'addition de dix nombres de 3–5 chiffres visuellement présentés. Sauf un sujet, tous peuvent calculer très rapidement sans abaque, probablement en comptant sur la représentation "intériorisée". Quelques-uns d'entre eux qui se trouvent au niveau intermédiaire de la maîtrise, remuent leurs doigts comme s'ils étaient en train de manipuler un abaque réel et la prohibition de ce mouvement ou l'interférence du frappement des doigts réduisent leur performance. Les deux meilleurs opérateurs ont marqué presque 100% sans augmentation de temps sous ces conditions. Ce fait suggère que leurs opérations mentales ne comportent en effet que la représentation visuelle sans compter sur les éléments moteurs.

Tous les sujets peuvent répondre aux questions simples sur des vérités ou des préférences pendant le calcul par abaque sans augmentation de temps ou d'erreurs. C'est peut-être parce que, après leur apprentissage excessif, les opérateurs n'ont besoin que d'attention limitée pour le calcul et le traitement linguistique entre peu dedans. Cependant, la réponse aux questions différentes sur les mathématiques est très difficile. L'étude supplémentaire sera nécessaire pour déterminer lequel entre deux facteurs, c'est-à-dire, la nature plus exigeante de la question mathématique ou sa similarité dans le traitement, rend les deux opérations incompatibles.